AMERICAN JOURNAL of PHYSICS

Analysis of light scattered by a capillary to measure a liquid's index of refraction

Zhihong You, Daya Jiang, Zhibo Hou, and Jinghua Xiao

Citation: Am. J. Phys. 80, 688 (2012); doi: 10.1119/1.4729607
View online: http://dx.doi.org/10.1119/1.4729607
View Table of Contents: http://ajp.aapt.org/resource/1/AJPIAS/v80/i8
Published by the American Association of Physics Teachers

Analysis of light scattered by a capillary to measure a liquid's index of refraction

Zhihong You, Daya Jiang, Zhibo Hou, and Jinghua Xiao[a)]
School of Science, Beijing University of Posts and Telecommunications, Beijing, China 100876

(Received 19 December 2011; accepted 1 June 2012)

A simple optical method to measure a liquid's refractive index using a capillary is experimentally demonstrated. The analysis and simulation of the intensity distribution of light scattered by the capillary are accomplished with a ray tracing method, and three intensity step points are obtained. By measuring the scattering angles of these step points, we can obtain the refractive index of the liquid in the capillary, as well as the refractive index and the diameter ratio of the capillary. With a laser, a spectrometer, and a capillary, we can easily perform an interesting undergraduate physics experiment. [http://dx.doi.org/10.1119/1.4729607]

I. INTRODUCTION

The refractive index is an important parameter for liquids. Accurate measurement of its value is of great significance to chemical, pharmaceutical, food, petroleum, and other industries. There are many methods and devices to measure the refractive index of a liquid in an undergraduate physics course, such as equal thickness interference,[1] a precision Fabry-Perot interferometer,[2] and geometrical optics methods that make use of the critical angle[3] or imaging.[4]

Recently, various approaches have been proposed to measure the refractive index of a liquid using a cylindrical capillary. In these approaches, light is incident normally on a capillary, and the refractive index of the liquid is obtained from various properties of the emergent light. For example, a cylindrical focusing method was introduced in Ref. 5, where the refractive index of the liquid was obtained by measuring the focusing ability of the capillary filled with liquids.[5] Though this method is based on a simple theory of optical focusing, it cannot measure the refractive index of a viscous liquid and is demanding on the precision of the experimental setup and measurements. Capillary interferometry, which is based on wave optics theory, is another approach to measure the refractive index of the liquid from interference patterns.[6–9] Although this approach can be used to measure the refractive index of a semitransparent liquid, it involves complicated calculations of optical length and a large amount of data pertaining to the interference patterns. In addition, both the cylindrical focusing method and the interferometry method rely on knowing the parameters of the capillary, such as the diameters and the refractive index, in advance.

In this paper, we provide a simple method to measure the refractive index of the liquid within a capillary from the angular intensity distribution of light scattered by the capillary. Using a ray tracing method, three pairs of intensity step points are obtained where the refractive index of the liquid is smaller than that of the capillary wall. We find that by measuring the scattering angles of these step points we can determine the refractive index of the liquid, as well as the diameter ratio and the refractive index of the capillary. We also perform an experiment to observe the scattering pattern and measure the angles of these three step points. As the wall of the capillary, which is made of glass, has a refractive index larger than that of most liquids, our method is quite general.

The rest of the paper is organized as follows. In Sec. II, the experimental setup and phenomenon are introduced. In Sec. III, we analyze the intensity distribution and step points using the ray tracing method and obtain the procedure for measuring the refractive index of the liquid. The experimental results are presented in Sec. IV. Finally, in Sec. V, we recommend some other significant applications and draw a conclusion.

II. EXPERIMENTAL SETUP AND PHENOMENON

The experimental apparatus is modified from a spectrometer, as shown in Fig. 1(a). A two-dimensional adjusting frame is placed in the middle of the objective stage. A capillary of about 1.0 mm diameter is placed on the adjusting frame perpendicularly. An observing screen with a scale is fixed on the arm of the spectrometer, and the arm can move circularly around the main axis of the spectrometer. In a dark room, a He-Ne laser (wavelength 632.8 nm) is positioned 4.25 m away from the capillary, and the diameter of the laser beam is about 9 mm. As the diameter of the capillary (1.0 mm) is much smaller than the width of the laser beam and the divergent angle of the laser beam is small enough (about 2 mrad), the laser beam incident on the capillary can be well approximated as a plane wave with negligible error to the result. With a stop placed 25 cm before the capillary, the diameter of the incident light is decreased to about 2 mm for the purpose of visualization, yet is still larger than the capillary diameter. As the capillary shuts out part of incident light, the corresponding area on the screen is dark, surrounded by a bright band of the incident light. To center the laser beam with respect to the capillary, we adjust the capillary until the dark spot is in the center of the bright band of the incident light. By rotating the observing screen, we can observe the intensity distribution of the scattered light at any angle and measure the scattering angle of a certain point with an accuracy of 1 arc min.

To observe the intensity distribution of the scattered light over the entire angle range, the capillary is placed at the center of a cylinder-shaped screen about 15 cm in diameter. A hole is drawn on the screen to let the incident light pass through the screen and reach the capillary. The intensity distribution of the light scattered by the capillary is obtained by a camera (see Fig. 2). The diameter ratio of the capillary is $r/R = 0.5229$ (r and R are the inner and outer radii of the capillary, respectively), measured by a reading microscope (JCD3 50 mm). The liquid inside the capillary is simethicone, with a refractive index of $n = 1.405$. The refractive index of the capillary wall is $n_0 = 1.471$, measured by an abbe-refractometer.

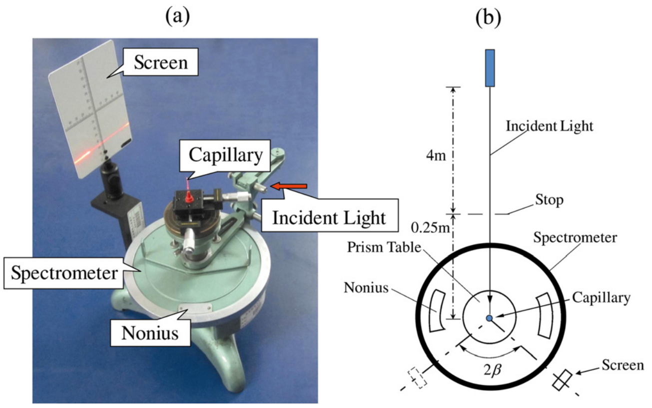


Fig. 1. (Color online) (a) Experimental setup and (b) schematic diagram.

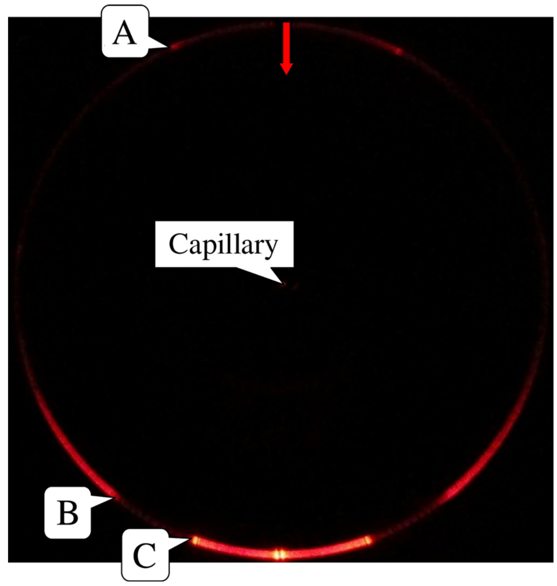


Fig. 2. (Color online) Intensity distribution of light scattered by the capillary. The diameter of the circular screen is 15 cm. The capillary parameters are $r/R = 0.5229$ and $n_0 = 1.471$, with simethicone inside. The scattering angles of A, B, and C are 154°55′, 37°41′, and 19°9′, which are measured from the incident direction.

As shown in Fig. 2, there are three pairs of intensity step points, located symmetrically on both sides of the incident light, and denoted as A, B, and C, respectively. Because of the symmetry, only the points on the left side are considered. In Sec. III, we will discuss the cause of these points and the relations between them and the structure (or parameters) of the capillary.

III. THEORETICAL ANALYSIS

A. Intensity distributions for the scattered light via ray tracing

The rays are divided into seven categories, as shown in Fig. 3. Because of the low reflectivity of glass, we consider only first-order reflection. Only the light incident on the right side is considered, because of the symmetry of the setup.

The reflected rays in Figs. 3(a)–3(d) can be regarded as background light because of their low relative intensities, continuous intensity distributions, and high divergences. This is not true for the light in Fig. 3(e), indicated as ray 1, which is also reflected light. Ray 1 cannot be ignored

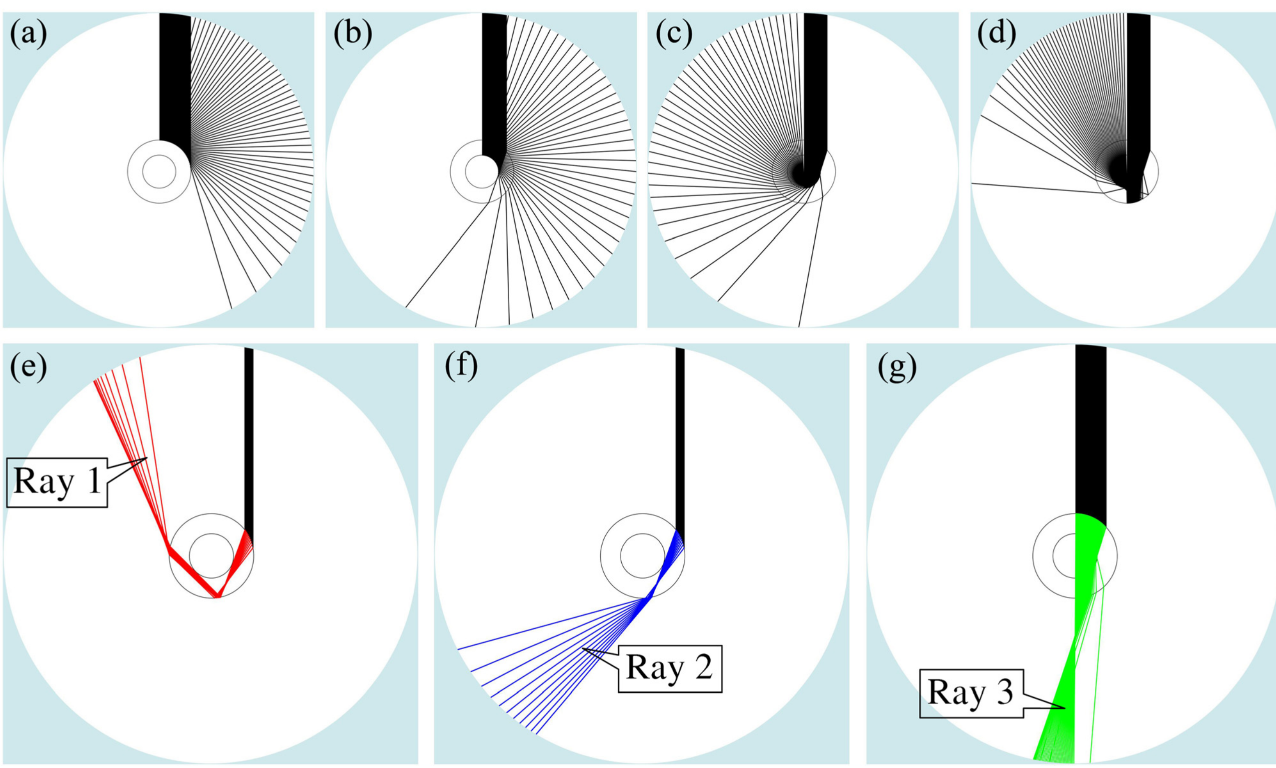


Fig. 3. (Color online) Seven categories of emergent rays. The parameters of the capillary are $r/R = 0.5229$, $n = 1.405$, and $n_0 = 1.471$. (a) Light reflected on the upper part of the outer wall. (b) Light reflected on the upper part of the inner wall. (c) Light reflected on the lower part of the inner wall. (d) Light passing through the core and reflected on the lower part of the outer wall. (e) Light reflected on the lower part of the outer wall but bypassing the core. (f) Transmitted light bypassing the core. (g) Transmitted light passing through the core.

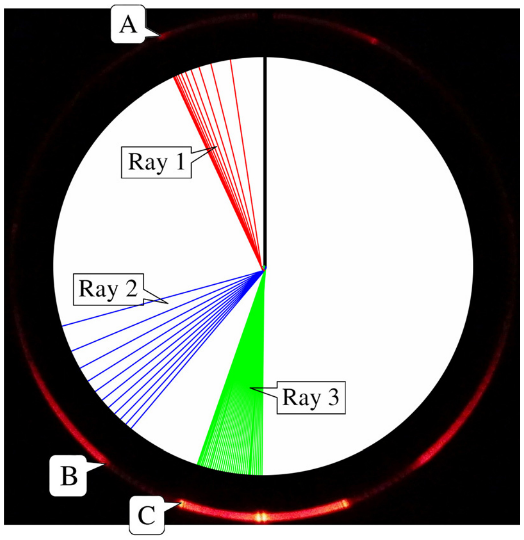


Fig. 4. (Color online) Comparison between the ray trajectories and the experimental intensity distribution. The parameters of the capillary are $r/R = 0.5229$, $n_0 = 1.471$, and the capillary is filled with simethicone.

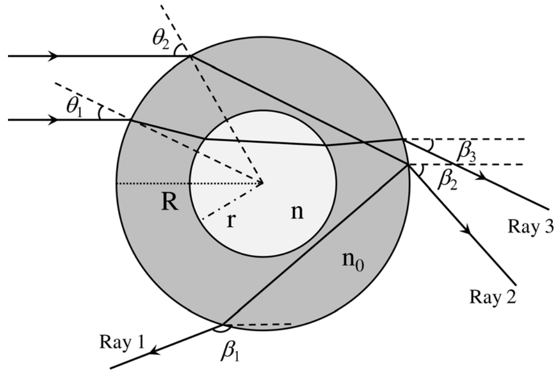


Fig. 5. Paths of rays 1, 2, and 3. The angles of incidence are θ_1 for the ray going through the liquid and θ_2 for the ray bypassing the liquid. The deviation angles are β_1, β_2, and β_3 for rays 1, 2, and 3.

because part of it converges at a certain angle, forming a high relative intensity in this area. The light rays shown in Figs. 3(f) and 3(g), indicated as rays 2 and 3, are the transmitted rays of high relative intensity and low divergence. They make a large contribution to the intensity distribution of the scattered light.

Figure 4 shows the trajectories of rays 1, 2, and 3, along with the experimental intensity distribution. The angle of incidence θ is defined as the angle between the incident ray and the normal of the capillary surface, while the angle of deviation β is defined as the angle from the emergent ray relative to the incident ray, measured clockwise. As the incident rays are of equal distance, ignoring the reflection and transmission, the density of the emergent rays can represent the intensity of emergent light. The intensity distribution of the scattered light is analyzed as follows:

(1) Ray 3 is of high density, which leads to a high intensity distribution in the corresponding area on the screen.
(2) The density of ray 2 decreases as the scattering angle increases; therefore, the intensity gradually becomes weaker with increasing scattering angle.
(3) Ray 1 converges in a certain angle range, where there is a bright band in the intensity distribution.
(4) The areas between rays 1 and 2, and rays 2 and 3, cannot be reached except by the background light. Thus, both areas in Fig. 4 are nearly dark.

Moreover, we can see that points A and B have the minimum deviation angles of rays 1 and 2, respectively, and point C has the maximum deviation angle of ray 3. To obtain the scattering angles of these three step points, we need to find the deviation angles of rays 1, 2, and 3 and then calculate their maximum or minimum, as we will do next.

B. The scattering angles of the three intensity step points

The paths of rays 1, 2, and 3 are illustrated in Fig. 5. The angles of incidence are θ_1 for the ray going through the liquid and θ_2 for the ray bypassing the liquid. The angles of deviation of the outgoing rays, measured clockwise with respect to the incident direction, are β_1, β_2, and β_3 for rays 1, 2, and 3, respectively. Applying Snell's law[10] and geometrical relations, we can easily obtain the three deviation angles in terms of the angle of incidence

$$\beta_1 = 2\theta - 4\arcsin\frac{\sin\theta}{n_0} + \pi, \qquad \arcsin\frac{rn_0}{R} < \theta < \frac{\pi}{2}, \tag{1}$$

$$\beta_2 = 2\left(\theta - \arcsin\frac{\sin\theta}{n_0}\right), \qquad \arcsin\frac{rn_0}{R} < \theta < \frac{\pi}{2}, \tag{2}$$

$$\beta_3 = 2\left(\theta - \arcsin\frac{\sin\theta}{n_0} + \arcsin\frac{R\sin\theta}{rn_0} - \arcsin\frac{R\sin\theta}{rn}\right), \quad 0 < \theta < \arcsin\frac{rn}{R}. \tag{3}$$

Here, θ represents either θ_1 or θ_2 as appropriate. We plot the three deviation angles against the angle of incidence in Fig. 6.

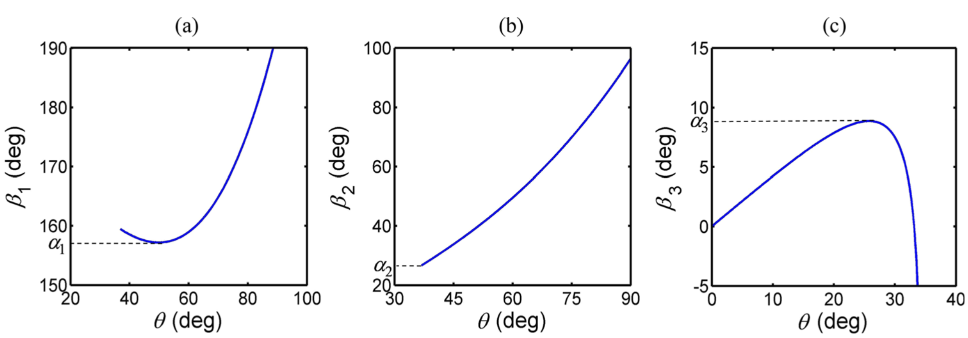


Fig. 6. The three deviation angles plotted vs the angle of incidence for (a) ray 1, (b) ray 2, and (c) ray 3, with $n_0 = 1.5$, $r/R = 0.4$, and $n = 1.4$.

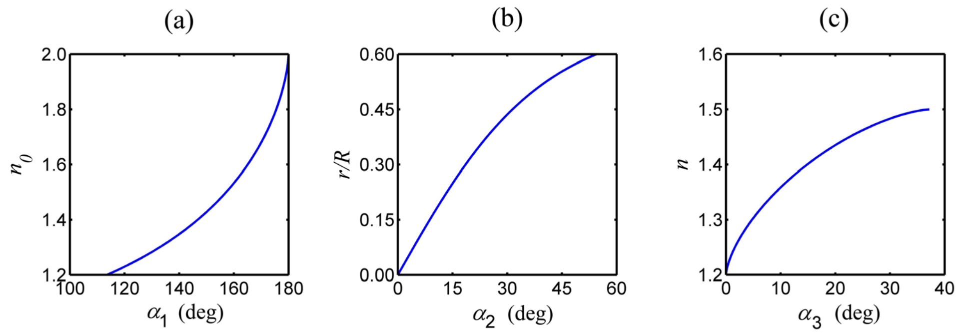


Fig. 7. Plots of the three step-point angles (horizontal axes): (a) α_1 vs n_0; (b) α_2 vs r/R, with $n_0 = 1.5$; (c) α_3 vs n, with $n_0 = 1.5$ and $r/R = 0.5$.

As we have seen above, β_1 has the minimum at point A. Figure 6(a) shows that β_1 does not change monotonically with respect to θ; there is instead a minimum α_1, which corresponds to point A on the screen. By solving

$$\frac{d\beta_1}{d\theta} = 0, \tag{4}$$

we can obtain θ_m, the angle of incidence that yields the minimum β_1

$$\theta_m = \arcsin\sqrt{\frac{4 - n_0^2}{3}}. \tag{5}$$

Inserting Eq. (5) into Eq. (1), we obtain α_1, the angle of point A

$$\alpha_1 = 2\arcsin\sqrt{\frac{4 - n_0^2}{3}} - 4\arcsin\sqrt{\frac{4 - n_0^2}{3n_0^2}} + \pi. \tag{6}$$

We can see from Eq. (6) that α_1 depends only on n_0; this function is plotted in Fig. 7(a). Thus, we can draw our first conclusion: By measuring the angle of point A, we can obtain the refractive index of the capillary wall.

The minimum of β_2 is denoted as α_2 in Fig. 6(b), which is the angle at point B. Since β_2 increases monotonically with θ, inserting the minimum of θ into Eq. (2) gives

$$\alpha_2 = 2\left(\arcsin\left(\frac{n_0 r}{R}\right) - \arcsin\left(\frac{r}{R}\right)\right). \tag{7}$$

Once n_0 is determined from Eq. (6), α_2 depends only on r/R. The curve of α_2 vs r/R (for $n_0 = 1.5$) is shown in Fig. 7(b). In this way, we can obtain the diameter ratio from the measurement of α_2. This is our second conclusion.

As shown in Fig. 6(c), β_3 doesn't change monotonically with θ, so point C corresponds to the maximum of β_3, indicated as α_3. Although it is complicated to obtain an analytic expression for α_3 from Eq. (3), we can still obtain the relation between α_3 and n numerically. The curve of α_3 vs n (for $n_0 = 1.5$ and $r/R = 0.5$) is shown in Fig. 7(c). From this curve, we draw our third conclusion: The refractive index of the liquid can be obtained by measuring α_3.

Several conditions in the experiment should be satisfied for a successful measurement. One of the conditions is that the diameter ratio r/R should be smaller than I/n_0 and $\sqrt{(4 - n_0^2)/(3n_0^2)}$. From Eq. (2), we can find that ray 2 cannot exist if r/R is greater than I/n_0. As all incident rays could not reach the liquid, and point B would disappear, this method would be invalid in this case. The minimum of β_1 can be obtained when θ_m falls in the range

$$\arcsin\frac{rn_0}{R} < \theta_m < \frac{\pi}{2}. \tag{8}$$

Under this condition, we can obtain

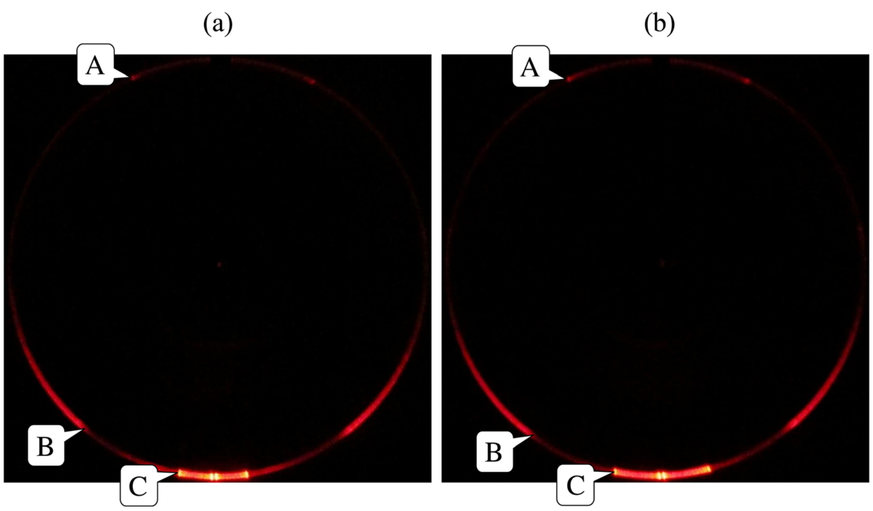


Fig. 8. (Color online) Intensity distributions of the scattered light. The diameter of the circular screen is about 15 cm. The parameters of the capillaries are (a) $r/R = 0.5189$, and $n_0 = 1.471$, filled with water, and (b) $r/R = 0.5224$, and $n_0 = 1.471$, filled with alcohol.

Table I. Angles of the intensity step points for three different liquids.

Liquid	Point A	Point B	Point C
Water	154°47′	37°33′	9°17′
Alcohol	154°25′	37°48′	12°42′
Simethicone	154°55′	37°41′	19°9′

$$\frac{r}{R} < \sqrt{\frac{4-n_0^2}{3n_0^2}}. \tag{9}$$

As the wall of the capillary is made of glass, its refractive index is generally larger than 1.3, and $1/n_0$ is larger than $\sqrt{(4-n_0^2)/(3n_0^2)}$, our theory requires that r/R be smaller than $\sqrt{(4-n_0^2)/(3n_0^2)}$.

Point C corresponds to the maximum deviation angle of ray 3. To find the maximum of β_3, one can start from

$$\frac{d\beta_3}{d\theta} = 2\left(1 - \frac{\cos(\theta)}{\sqrt{n_0^2 - \sin^2(\theta)}} + \frac{\cos(\theta)}{\sqrt{(rn_0/R)^2 - \sin^2(\theta)}} - \frac{\cos(\theta)}{\sqrt{(rn/R)^2 - \sin^2(\theta)}}\right). \tag{10}$$

From this equation, we can see that β_3 decreases monotonically with increasing angle of incidence when n is "small enough," or increases monotonically when n is "large enough." In both cases, the intensity distribution of ray 3 varies continuously with scattering angle and does not produce an intensity step. Therefore, point C exists if and only if β_3 has one maximum, which can be described by the following inequalities:

$$\frac{d\beta_3}{d\theta} \geq 0 \qquad (\text{minimum } \theta), \tag{11}$$

$$\frac{d\beta_3}{d\theta} \leq 0 \qquad (\text{maximum } \theta). \tag{12}$$

In the case $n < n_0$, the minimal and maximal angles of incidence are 0 and arcsin (rn/R), respectively. Inserting these into Eqs. (10)–(12), we obtain

$$\frac{d\beta_3}{d\theta} = 2\left(1 - \frac{1}{n_0} + \frac{R}{rn_0} - \frac{R}{rn}\right) \geq 0 \qquad (\theta = 0), \tag{13}$$

$$\frac{d\beta_3}{d\theta} = -\infty \leq 0 \qquad (\theta = \arcsin(rn/R)). \tag{14}$$

A critical value of the liquid refractive index, denoted as n_1, can be obtained from Eq. (13)

$$n_1 = \frac{1}{r/R - r/Rn_0 + 1/n_0}. \tag{15}$$

Note that point C exists only when the refractive index of the liquid is greater than n_1.

When $n > n_0$, the minimal and maximal angles of incidence are 0 and arcsin (rn_0/R), respectively. Inserting these into Eqs. (10)–(12), we can obtain the conditions for the existence of point C

$$\frac{d\beta_3}{d\theta} = 2\left(1 - \frac{1}{n_0} + \frac{R}{rn_0} - \frac{R}{rn}\right) \geq 0 \qquad (\theta = 0), \tag{16}$$

$$\frac{d\beta_3}{d\theta} = +\infty \leq 0 \qquad (\theta = \arcsin(rn_0/R)). \tag{17}$$

Now we can see that Eq. (17) cannot be satisfied, which indicates that point C will disappear when $n > n_0$.

In conclusion, the range of the liquid refractive index that can be measured using this approach is

$$\frac{1}{r/R - r/Rn_0 + 1/n_0} < n < n_0. \tag{18}$$

When the capillary parameters are $n_0 = 1.471$ and $r/R = 0.52$, the measurable n is between 1.18 and 1.471, basically covering most common liquids.

IV. EXPERIMENTAL RESULTS AND ANALYSES

In addition to measuring the angles of the three step points for simethicone (Fig. 2), we also measured the angles of the step points for other liquids. Photographs of the experimental light patterns are shown in Fig. 8. By averaging the readings of step points on both sides, we can partly eliminate the error caused by the asymmetry of the capillary. The angles of the intensity step points in Figs. 2 and 8 are shown in Table I, and the final physical parameters obtained are shown in Table II.

In Table II, n_0, r/R, and n are calculated from the angles shown in Table I; n_0' and n' are obtained using an abbe-refractometer; and r'/R' is obtained using a JCD3 50 mm reading microscope. As shown in Table II, the differences in the measured parameters between our method and the traditional methods are smaller than 0.5%.

The uncertainty of the results in this experiment mainly comes from the determination of step points. In fact, these points are not strict points, but small light spots, which make the scattering angles vary over a small range. Another important source of uncertainty is the asymmetry of the capillary, since all our analysis is based on the assumption of perfect cylindrical geometry, which cannot be fully realized.

Table II. Physical parameters obtained from Table I.

Liquid	n_0	n_0'	Error (%)	r/R	r'/R'	Error (%)	n	n'	Error (%)
Water	1.4749	1.4710	0.27	0.5201	0.5189	0.23	1.3299	1.3330	0.23
Alcohol	1.4712	1.4710	0.01	0.5248	0.5224	0.46	1.3559	1.3616	0.42
Simethicone	1.4762	1.4710	0.35	0.5203	0.5229	0.49	1.4063	1.4058	0.04

V. SUMMARY

The measurement of the refractive index of a liquid inside a capillary is easily performed. The laser, spectrometer, and capillary in the experiment are available in an undergraduate laboratory. The analysis requires only the knowledge of geometrical optics and can be easily understood by undergraduates. By simply measuring the scattering angles of intensity step points instead of interference fringes, we can easily obtain many parameters. Furthermore, because only a small amount of liquid is needed during the measurement, this method enables us to measure the refractive index of poisonous or flammable liquids, which cannot be done using traditional methods. From the measurement of the refractive index, we can go on to obtain some other properties of liquids related to the refractive index.

ACKNOWLEDGMENTS

This work is supported by the National College Innovation Program of Beijing University of Posts and Telecommunications. The authors would like to thank Dr. Hujiang Yang and Dr. Jianjun Chen for their beneficial suggestions.

[a]Electronic mail: jhxiao@bupt.edu.cn

[1]G. R. Davies, "Refractive index of a liquid by wedge-film interference," Am. J. Phys. **44**(12), 1215–1216 (1976).

[2]M. Deng, C. Tang, T. Zhu *et al.*, "Refractive index measurement using photonic crystal fiber-based Fabry–Perot interferometer," Appl. Opt. **49**(9), 1593–1598 (2010).

[3]E. Richard and V. Keuren, "Refractive index measurement using total internal reflection," Am. J. Phys. **73**(7), 611–614 (2005).

[4]K. Tennekone, "A simple, sensitive method for measuring the refractive index of a liquid," Am. J. Phys. **44**(3), 303–304 (1976).

[5]S. Calixto *et al.*, "Capillary refractometer integrated in a microfluidic configuration," Appl. Opt. **47**(6), 843–848 (2008).

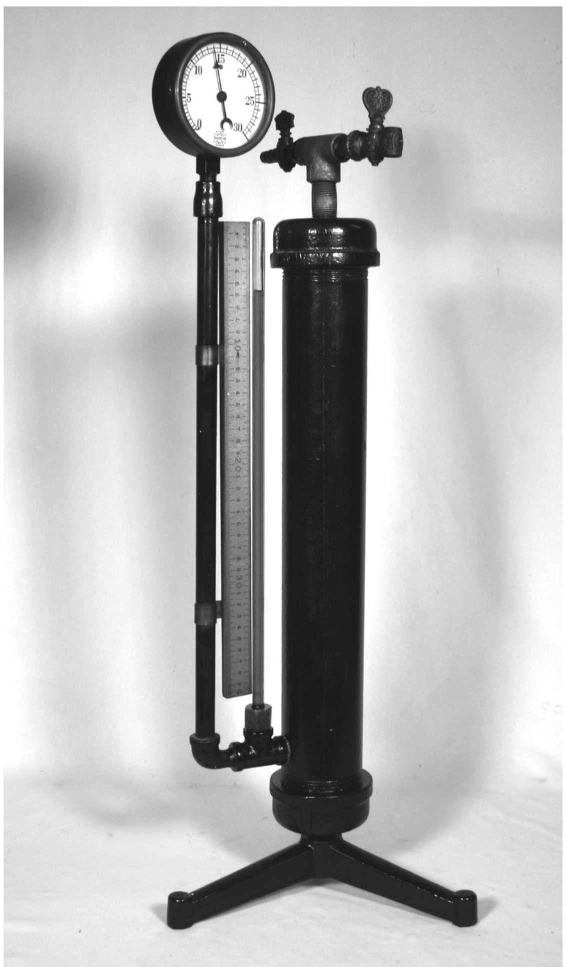
[6]B. Krattiger *et al.*, "Laser-based refractive-index detection for capillary electrophoresis: Ray-tracing interference theory," Appl. Opt. **32**(6), 956–965 (1993).

[7]A. Yang *et al.*, "Measuring the refractive indices of liquids with a capillary tube interferometer," Appl. Opt. **45**(31), 7993–7998 (2006).

[8]S. Qi *et al.*, "Analysis of capillary interferometry for measuring refractive indices of minute samples," Appl. Opt. **43**(3), 530–536 (2004).

[9]Z. Hou, X. Zhao, and J. Xiao, "A simple double-source model for interference of capillaries," Eur. J. Phys. **33**, 199–206 (2012).

[10]M. Born and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge University Press, Cambridge, UK, 2002), pp. 132–133.



Boyles Law Apparatus. This is listed in a 1921 Cenco bulletin as being designed by a High School Teacher for high school teachers. The glass tube, partly filled with oil, is connected at the bottom to the iron cylinder, also containing oil. The scale beside the tube gives the volume of the air trapped in the glass tube and the dial gauge records the pressure at the bottom of the tube (no correction is made for the head of oil in the tube). The pressure is then modified with a vacuum or pressure pump, and a graph of the pressure as a function of the reciprocal of the volume is a straight line. The apparatus was designed by Walter R. Ahrens of Englewood High School in Chicago and cost $25.00. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College.)